# Stirrup Cages and Ladders as Shear Assembly=Dangerous Arrangement of Shear Reinforcement

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*Published in:*
Workshop: Development of EN 1992 in Relation to New Research Results and to the CEB-FIP Model Code 1990

*Publication date:*
1994

*Document Version*
Early version, also known as pre-print

# Stirrup cages and ladders as shear assembly = Dangerous arrangement of shear reinforcement

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# Stirrup cages and ladders as shear assembly = Dangerous arrangement of shear reinforcement?

## 1.1 Introduction

According to ENV 1992-1-1: 1991 "Design of Concrete Structures – Part 1: General Rules and Rules for Buildings" /1/ the shear capacity of a beam can be determined by using two different methods: the *Standard Method* (SM) and the *Variable Truss Angle Method* (VTAM)
In the following the theoretical values of the shear capacity calculated after these methods will be compared with test results from two test series, test series 1 and test series 2.

## 1.2 Test set up, beam types and identification

Test series 1 deals with beams with shear reinforcement in the shape of open stirrups and ladders as shown in figure 1, and described in
*Forschungsbericht, Rationalisierung der Bewehrungstechnik im Stahlbetonbau, TI 4, Vereinfachte Schubbewehrung in Balken, T1261/4, IRB Verlag, Universität Stuttgart, Dez. 1979 [2].*
Test series 2 deals with beams with shear reinforcement in the shape of closed stirrups as shown in figure 2 (figure 2.2.2 in /3/) and described in
*Shear Tests on Reinforced Concrete T-beams, Series T. Report No. R72, 1976. Structural Research Laboratory, Technical University of Denmark* /3/
The schematic test set up and the cross section of the beams are shown in figure 3. The geometrical data are shown in table 1. – The material data of the beams are shown in the table 2.

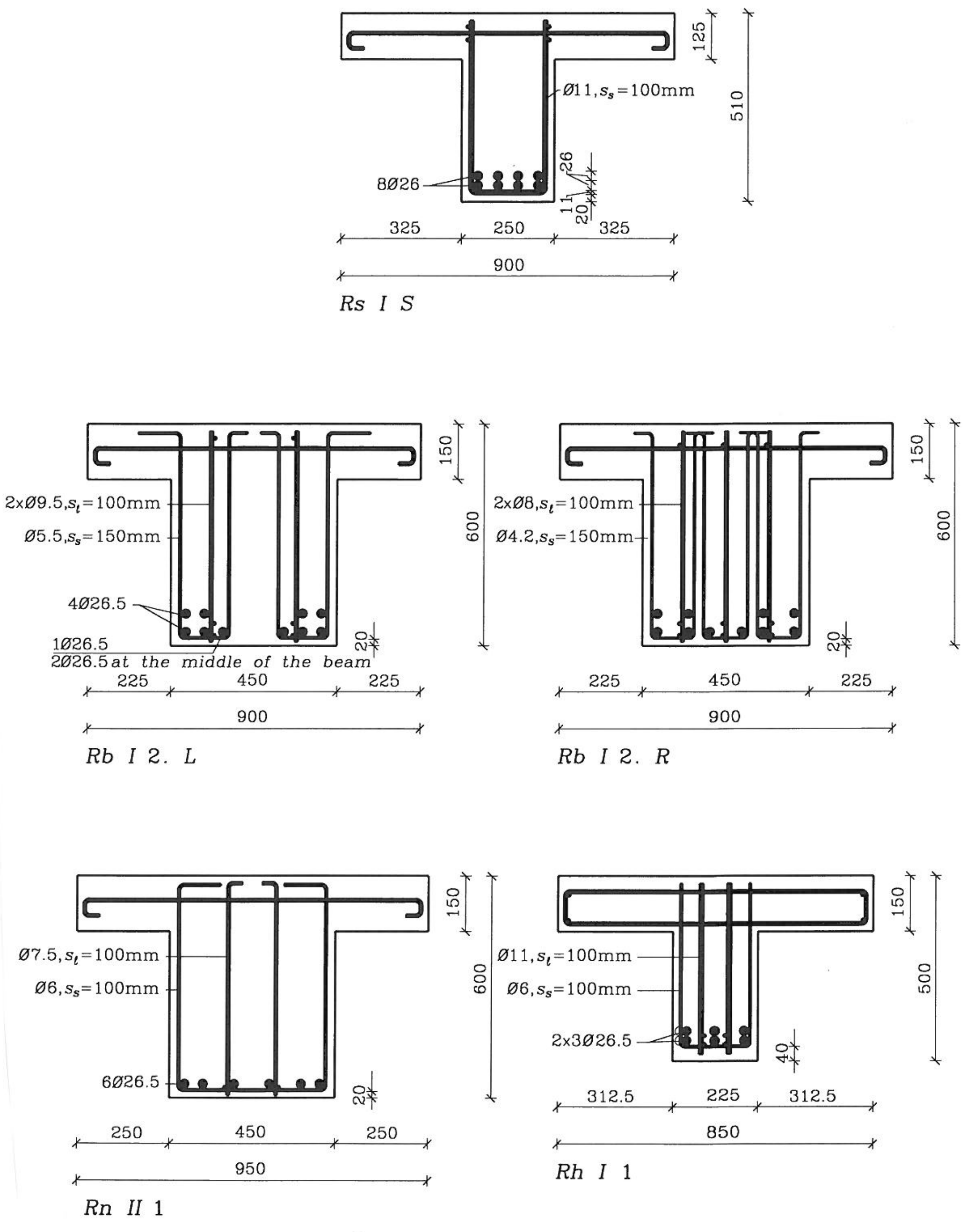
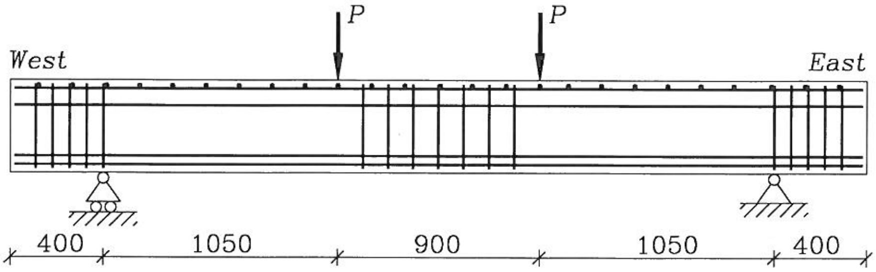
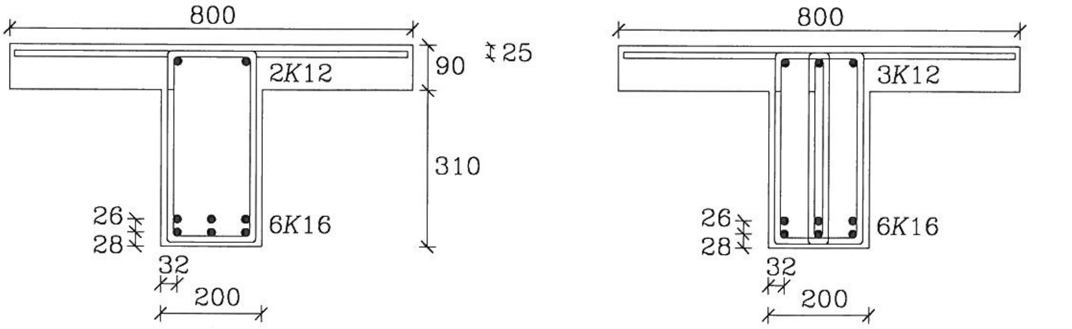



Figure 1. Types of shear reinforcement used in test series 1.
$L$ and $R$ refer to the left and the right part of the beam.




*Elevation of test beams*
*Variable stirrup reinforcement not shown*




*Testspecimen T6005* *Testspecimen T6010, T6022, T6032*

*Normal sections of test beam*

*T6005 and T6010: diameter and spacing of stirrups 6 and 131 mm resp.*
*T6022 : diameter and spacing of stirrups 10 and 175 mm resp.*
*T6032 : diameter and spacing of stirrups 10 and 117 mm resp.*
*With the exception of the beam T6005, the shear reinforcement was placed as double stirrups.*

Figure 2. Types of shear reinforcement used in test series 2.

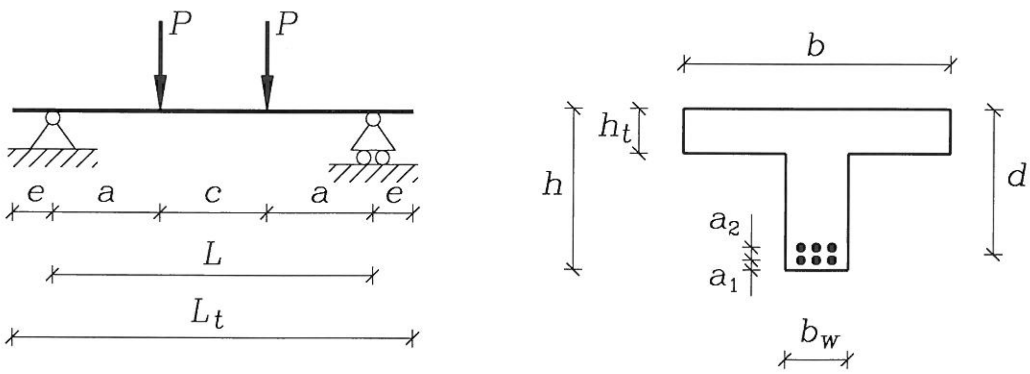



Figure 3. Schematic test set up and cross section of the beams.

## 1.3 Theory for caluculation of the theoretical value of the shear capacity of the beams

### 1.3.1 Determination of the equivalent values of the spacing of stirrups $s_{sq}$ and the area of stirrups $A_{sw,seq}$

In table 2 it is seen that the material used for stirrups and ladders has different yield strength $f_{y,s}$ and $f_{y,l}$, respectively, and furthermore, in the beam RbI2,L the spacing of the stirrups and the ladders is different. L refers to the left part of the beam.

Therefore, by using the *Standard Method* (SM) and the *Variable Truss Angle Method* (VTAM) for calculation of the shear capacity for the beams it is convenient to determine an equivalent spacing $s_{eq}$ for the combination of stirrups and ladders when using an area of the shear reinforcement $A_{sw,l\,eq} = A_{sw,s}$ per layer, equal to the cross sectional area $A_{sw,s}$ of the stirrups per layer used in the beam.

Due to the difference in the yield strength of the stirrups and the ladders the equivalent area of the ladders is to be determined from

$$A_{sw,l\,eq} = A_{sw,l}\,\frac{f_{y,l}}{f_{y,s}} \qquad \text{where} \tag{1}$$

| | |
|---|---|
| $A_{sw,l\,eq}$ | is the equivalent area of ladders in one layer corresponding to the yield strength $f_{y,s}$ of the stirrups |
| $A_{sw,l}$ | is the area of the ladders in one layer |
| $f_{y,l}$ | is the yield strength of the ladders |
| $f_{y,s}$ | is the yield strength of the stirrups |

Now, the equivalent spacing of stirrups $s_{eq}$ is to be calculated as shown below

$$A_{sw,s\,eq}/s_{eq} = A_{sw,s}/s_{eq} = A_{sw,s}/s_s + A_{sw,l\,eq}/s_l \tag{2}$$

by inserting (1) and isolating $s_{eq}$

$$s_{eq} = \frac{s_s}{1 + A_{sw,l}f_{y,l}s_s/(A_{sw,s}f_{y,s}s_l)} \qquad \text{where} \tag{3}$$

| | |
|---|---|
| $A_{sw,s}$ | is the area of stirrups in one layer |
| $A_{sw,s\,eq}$ | is the equivalent area of stirrups in one layer corresponding to the combination of stirrups and ladders. In this investigation $A_{sw,s\,eq} = A_{sw,s}$ |
| $s_l, s_s$ | is the spacing of the ladders and the stirrups, respectively. |
| $s_{eq}$ | is the equivalent spacing of layers of stirrups according to the combination of ladders and stirrups. |

The equivalent amount of the shear reinforcement is described by

the ratio of shear reinforcement $\varphi_{sw} = A_{sw,s}/\left(b_w s_{eq}\right)$ (4)

and the mechanical degree of shear reinforcement $\Phi_{sw} = A_{sw,s} f_{y,s}/\left(b_w s_{eq} f_{c,\mathrm{cyl}}\right)$ (5)

where $b_w$ is the web width of the beams

$f_{c,\mathrm{cyl}}$ is the mean value of the uniaxial compressive cube strength of the concrete at the time of the test.

In the same way the amount of the longitudinal reinforcement is described by

the ratio of longitudinal reinforcement $\varphi = \dfrac{A_s}{bd}$ (6)

and the mechanical degree of longitudinal reinforcement $\Phi = A_s f_y/\left(bd f_{c,\mathrm{cyl}}\right)$ (7)

where $b$ is the flange width of the beams

$d$ is the effective depth of the cross section

$f_y$ is the yield strength of the longitudinal reinforcement

The data of the concretes, the longitudinal and shear reinforcement are shown in table 2.

### 1.3.2 Determination of the theoretical values of the shear capacity of the beams in the investigation

The determination of the theoretical values of the shear capacity of the beams are carried out by using the *Standard Method* (SM) and the *Variable Truss Angle Method* (VTAM) described in section 4.3.2 in /1/.

In these investigations the lever arm of internal forces $z$ is calculated as the distance between the centres of compression and tension zone using a constant value of the compressive stresses equal to $f_{c,\mathrm{cyl}}$ and a depth of the zone with the constant compressive stresses equal to $0.8x$, where $x$ is the neutral axis depth, see figure 4.4 in /1/.

#### 1.3.2.1 The Standard Method (SM)

The shear capacity in accordance with SM is calculated as the minimum value of

$$V_{u,sm} = \min \begin{cases} 1.5\tau_{Rd} k\left(1.2 + 40\rho_1\right) b_w d + \dfrac{A_{sw}}{s_{eq}} z f_{y,s} \\ \dfrac{1}{2} \nu f_{c,\mathrm{cyl}} b_w z \end{cases} \tag{8}$$

where the factor 1.5 is the partial safety factor for the basic design shear strength $\tau_{Rd}$ used in table 4.8 in /1/.

$\tau_{Rd}$ is the basic design shear strength

$k$ is a constant relating to section depth and curtailment. In these investigations $k = 1.6 - d \not< 1$ because lesser than 50% of the bottom reinforcement in the beams are curtailed.

$\rho_1$ $= A_s/(b_w d) \not> 0.02$, is the reinforcement ratio corresponding to $A_s$.

$\nu$ $= 0.7 - f_{c,\mathrm{cyl}}/200 \not< 0.5$, is the effectiveness factor on the concrete strength in the struts.

| | Test specimen | a mm | c mm | e mm | L mm | $L_t$ mm | $b_w$ mm | b mm | d mm | $h_t$ mm |
|---|---|---|---|---|---|---|---|---|---|---|
| Test series 1 | Rs I S | 1350 | 900 | 150 | 3600 | 3900 | 250 | 900 | 453 | 125 |
| | Rb I 2 | 1600 | 1400 | 150 | 4600 | 4900 | 450 | 900 | 544 | 150 |
| | Rn II 1 | 1600 | 1600 | 510 | 4800 | 5820 | 450 | 950 | 561 | 150 |
| | Rh I 1 | 1350 | 1350 | 200 | 4050 | 4450 | 225 | 850 | 427 | 150 |
| Test series 2 | T6005 | 1050 | 900 | 400 | 3000 | 3800 | 200 | 800 | 359 | 90 |
| | T6010 | 1050 | 900 | 400 | 3000 | 3800 | 200 | 800 | 359 | 90 |
| | T6022 | 1050 | 900 | 400 | 3000 | 3800 | 200 | 800 | 359 | 90 |
| | T6032 | 1050 | 900 | 400 | 3000 | 3800 | 200 | 800 | 359 | 90 |

Table 1: Measurements in the test set up and the cross-sections of the beams.

#### 1.3.2.2 The Variable Truss Angle Method (VTAM)

The shear capacity in accordance with VTAM for $\Phi_{sw} \leq \frac{1}{2}\nu$ is calculated from

$$V_{u,vta} = V_{Rd2} = b_w z \nu f_{c,\mathrm{cyl}}/(\cot\theta + tg\theta) \tag{9}$$

if the stresses in the concrete struts are decisive and

$$V_{u,vta} = V_{Rd3} = \frac{A_{sw}}{s_{eq}} z f_{y,s} \cot\theta \tag{10}$$

if the shear reinforcement is decisive.

The optimum value of $\cot\theta$ is found by settting $V_{Rd2} = V_{Rd3}$

$$b_w z \nu f_{c,\mathrm{cyl}}/(\cot\theta + \tan\theta) = \frac{A_{sw}}{s_{eq}} z f_{y,s} \cot\theta \tag{11}$$

by introducing the mechanical degree of shear reinforcement $\Phi_{sw} = \dfrac{A_{sw,s} f_{y,s}}{b_w s_{eq} f_{c,\mathrm{cyl}}}$

the optimum value of $\cot\theta$ is found

$$\cot\theta = \sqrt{\frac{\nu - \Phi_{sw}}{\Phi_{sw}}} \tag{12}$$

If $\Phi_{sw} > \frac{1}{2}\nu$ the shear capacity is calculated from

$$V_{u,vta} = \frac{1}{2} b_w z \nu f_{c,\mathrm{cyl}} \tag{13}$$

### 1.3.2.3 Test results

In /2/ the test results in test series 2 are given by $\tau_{ou} = V_{u,\mathrm{test}}/(b_w z)$. To compare with the theoretical results $V_{u,\mathrm{test}}$ is calculated as $V_{u,\mathrm{test}} = \tau_{ou} b_w z$.

The theoretical values of the shear capacity determined by use of the methods VTAM and SM, respectively, $V_{u,vta}$ and $V_{u,sm}$ together with the test result $V_{u,\mathrm{test}}$ are shown in table 2.

In figures 4 and 5 the relationships $V_{u,\mathrm{test}}/V_{u,vta}$ and $V_{u,\mathrm{test}}/V_{u,sm}$ between the shear capacity found by test $V_{u,\mathrm{test}}$ and the theoretical values of the shear capacities determined by use of the methods VTAM and SM are shown.

In table 2 it is seen that for values of $\Phi_{sw}$ lesser than about 0.3 the VTAM-method gives substantial higher values of the theoretical shear capacity compared with the values from the SM-method.

In table 2 and figure 4 it is seen that for values of $\Phi_{sw}$ lesser than about 0.3 the theoretical determined value of the shear capacity calculated by use of the VTAM-method is smaller than the shear capacity found by test, when closed stirrups are used and higher if open stirrups, stirrup cages and ladders are used.

In table 2 and figure 5 it is seen that for values of $\Phi_{sw}$ lesser than about 0.3 the theoretical values of the shear capacity calculated by use of the SM-method are smaller than the shear capacity found by test for all types of shear reinforcement, except for the beam RbI2, and it is seen that for beams with closed stirrups the theoretical values of the shear capacity are substantial smaller than the values found by tests.

For values of $\Phi_{sw}$ higher than about 0.3 the theoretical values of the shear capacity determined of the VTAM- and the SM-method are identical, because for these values of $\Phi_{sw}$ the strength of the concrete is decisive.

| | Test specimen | $f_{c,cyl}$ at time of test MPa | Yield strength of stirr-ups $f_{y,s}$ MPa | Yield strength of lad-ders $f_{y,l}$ MPa | Equiva-lent spacing of layers of stir-rups $s_{eq}$ mm | Mech. degree of shear reinfor-cement $\Phi_{sw}$ | $V_{u,vta}$ kN | $V_{u,sm}$ kN | $V_{u,test}$ kN |
|---|---|---|---|---|---|---|---|---|---|
| Test series 1 | Rs I S | 25.9 | 560 | - | 100.0 | 0.164 | 686.4 | 556.4 | 578.5 |
| | Rb I 2,L | 28.2 | 598 | 531 | 30.2 | 0.148 | 1251.4 | 1003.3 | 867.2 |
| | Rb I 2,R | 28.2 | 570 | 532 | 24.7 | 0.151 | 1263.0 | 1020.5 | 990.6 |
| | Rn II 1,L | 20.4 | 562 | 528 | 40.5 | 0.085 | 932.2 | 559.0 | 703.5 |
| | Rn II 1,R | 20.4 | 562 | 528 | 40.5 | 0.085 | 932.2 | 559.0 | 659.7 |
| | Rh I 1,L | 21.2 | 574 | 633 | 21.2 | 0.320 | 489.4 | 490.9 | 563.0 |
| | Rh I 1,R | 21.2 | 574 | 633 | 21.2 | 0.320 | 489.4 | 490.9 | 539.7 |
| Test series 2 | T6005 | 10.58 | 323 | | 131 | 0.066 | 126.3* | 82.5 | 157.9 |
| | T6010 | 10.30 | 323 | | 131 | 0.135 | 164.3 | 124.4 | 190.2 |
| | T6022, L | 10.35 | 321 | | 175 | 0.278 | 201.4 | 203.4 | 234.4 |
| | T6022, R | 10.35 | 321 | | 175 | 0.278 | 201.4 | 203.4 | 219.4 |
| | T6032, L | 11.75 | 321 | | 117 | 0.367 | 235.6 | 235.6 | 238.3 |
| | T6032, R | 11.75 | 321 | | 117 | 0.367 | 235.6 | 235.6 | 240.3 |

Table 2: Test data. Unaxial compressive cylinder strength at time of test $f_{c,cyl}$, yield strength of stirrups and ladders $f_{y,s}$ and $f_{y,l}$ respectively, the equivalent spacing of layers of stirrups $s_{eq}$, the mechanical degree of shear reinforcement $\Phi_{sw} = A_{sw,s}\ f_{y,s}/(b_w\ s_{eq}\ f_{c,cyl})$, the theoretical dertermined values of shear capacity $V_{u,vta}$ and $V_{u,sm}$ respectively and the shear capacity found by tests $V_{u,test}$.

* $V_{u,vta} = 126.3$ kN is calculated using $\cot\Theta = 2{,}971$ (the theoretical optimal value). Using $\cot\Theta = 2{,}5$ the max. value acording to /1/ is $V_{u,vta} = 106{,}3$ kN.

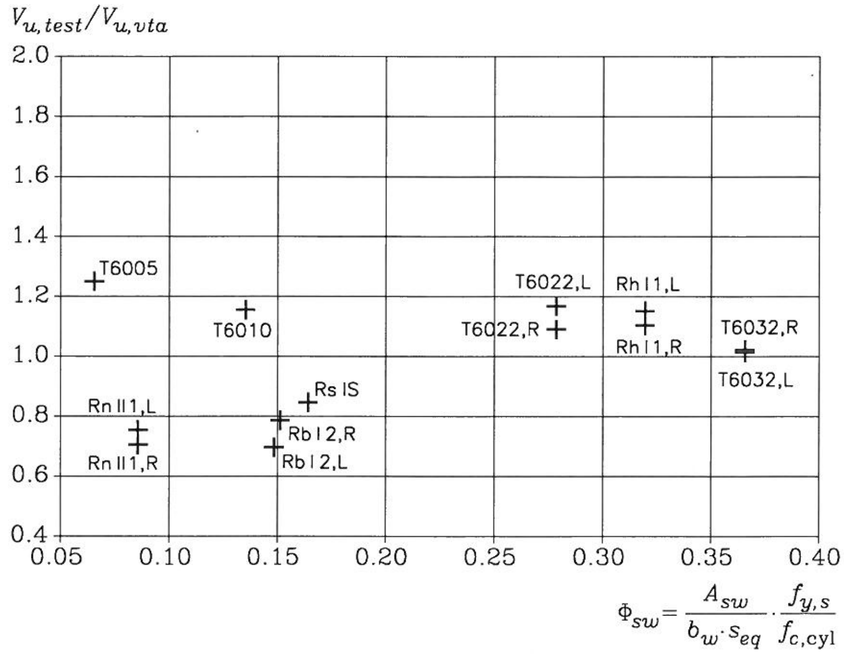



Figure 4: $V_{u,\text{test}}/V_{u,vta} - \Phi_{sw}$ relationsship for the beams in test series 1 and 2.

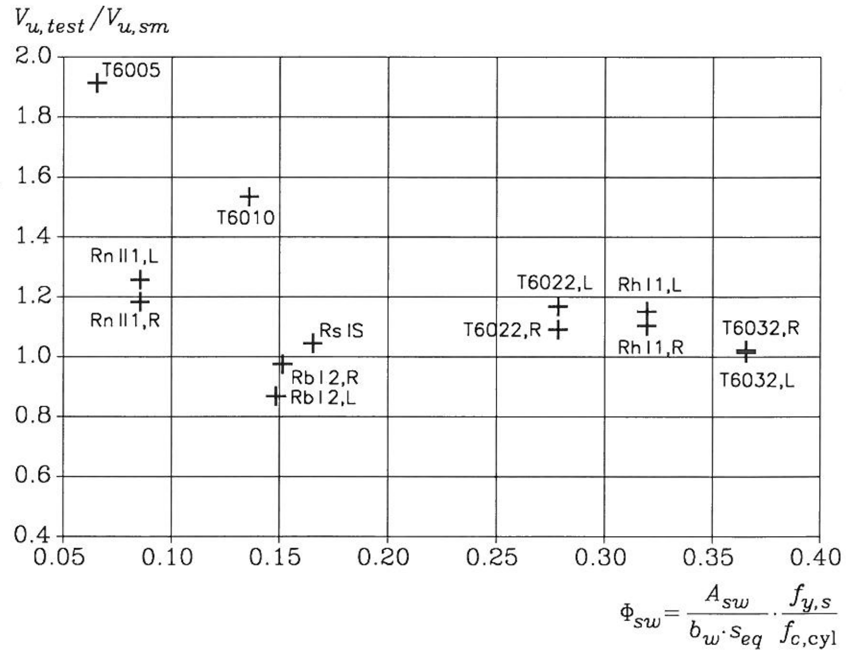



Figure 5: $V_{u,\text{test}}/V_{u,sm} - \Phi_{sw}$ relationship for the beams in test series 1 and 2.

### 1.4 Conclusion

For values of $\Phi_{sw}$ lesser than 0.3 it is found above that using shear reinforcement in the shape of open stirrups and ladders as shown in figure 1 the test results are lesser than the theoretical values determined by use of the VTAM-method, while using closed stirrups as shown in figure 2 the test results are higher than the theoretical values of the shear capacity.

These results are interpreted as the anchorage of the shear reinforcement in the shape of open stirrups, and ladders are not sufficient. By using closed stirrups the VTAM-method will give values of the shear capacity in accordance with the test results while the results determined by use of the SM-method with the exception of the beam $Rb\,I2$ will be substantial lesser than the test results if $\Phi_{sw} < 0.3$.

For $\Phi_{sw}$ larger than about 0.3 the two methods will give exactly the same result.

The test described in /2/ shows that the anchorage of shear reinforcement in the shape of open stirrups, stirrup cages and ladders as shown in figure 5.14 in /1/ is not sufficient for full utilization of the material of the shear reinforcement. The tests described in /2/ show that using the *The Variable Truss Angle Method* for calculating the shear capacity for shear reinforcement in the shape of open stirrups, stirrup cages and ladders it is necessary to introduce an efficiency factor $\alpha_s$ on the yield stress of the shear reinforcement. Due to the relative few tests carried out with shear reinforcement in the shape of open stirrups, stirrup cages and ladders it is not possible here to give a value for $\alpha_s$, but the tests in /2/ seem to show that the value of $\alpha_s$ shall be in the interval $0.7 < \alpha_s < 0.85$. Additional test has to be carried out to determine values for $\alpha_s$ in different cases.

### References


/1/ ENV 1992-1-1: 1991 "Eurocode 2: Design of Concrete Structures – Part 1: General rules and rules for buildings".

/2/ Forschungsbericht, Rationalisierung der Bewehrungstechnik im Stahlbetonbau, TI 4. Vereinfachte Schubbewehrung in Balken, T1261/4, IRB Verlag, Universität Stuttgart, Dez. 1979.

/3/ Shear Tests on Reinforced Concrete T-beams, Series T. Report No. R 72, 1976. Structural Research Laboratory, Technical University of Denmark.